<u>Minimal Fractal Manifold and the Dual Nature of Dark Matter</u>

Ervin Goldfain

Abstract

As of today, the nature of Dark Matter (DM) remains controversial. Broadly speaking, there are two major schools of thought on the topic. While the first school considers DM to be an extrapolation of particle physics, the second one asserts that DM is a signature of gravitational phenomena. The basic tenet of the second school of thought is that DM can be explained away either through revisions of General Relativity (GR), through its inherent nonlinearity or by deploying non-traditional interpretations of GR. The goal of this brief note is to point out that DM and gravitational physics may no longer be mutually exclusive, but complementary manifestations of the same reality. Elaborating from the minimal fractality of spacetime above the Fermi scale - along with its quaternion formulation - we suggest that DM can replicate the contribution of gravitational nonlinearities on galactic scales.

Key words: Dark Matter, General Relativity, Minimal Fractal Manifold, Cantor Dust.

"Contraria non contradictoria, sed complementa sunt"

Niels Bohr

<u>1. Introduction</u>

Rooted in the Renormalization Group program of Quantum Field Theory (QFT), the *minimal fractal manifold* (MFM) describes a spacetime continuum having arbitrarily small deviations from four-dimensions ($\varepsilon = 4 - D << 1$). This fine structure of spacetime is conjectured to set in near or above the Fermi scale. The emergence of the MFM sheds light on many ongoing puzzles of the Standard Model (SM), while meeting all consistency requirements mandated by effective QFT in the conformal limit $\varepsilon = 0$. The underlying rationale, conceptual benefits and implications of the MFM for the development of QFT and SM are reported in [1-12].

Recently, a proposal was put forward according to which DM represents *Cantor Dust*, a large-scale dimensional phase produced by MFM condensation in the early Universe [13-15]. Here we show that the quaternion formulation of the MFM may bridge the gap between Cantor Dust and the gravitational interpretation of DM. At the end of the day, the debate over the nature of DM may turn out to be a vivid illustration of *duality* in theoretical physics.

2. Quaternion representation of the MFM

If component deviations along the four spacetime coordinates are assumed to be independent from each other, the overall deviation ε amounts to the sum [12]

$$\varepsilon = 4 - D = \sum_{\mu} \varepsilon_{\mu} = \sum_{\mu} (1 - D_{\mu}), \ \ \mu = 0,1,2,3 \tag{4}$$

Using the change of variables,

$$\varepsilon^{1/2} = \xi, \ \ \varepsilon_{\mu}^{1/2} = \xi_{\mu} \tag{5}$$

opens the possibility of casting the MFM formalism in the language of quaternions. The *dimensional quaternion* (q) and its conjugate $(\bar{q})$ are accordingly defined as

$$\boxed{q = \xi_0 + i\xi_1 + j\xi_2 + k\xi_3} \tag{6a}$$

$$\boxed{\bar{q} = \xi_0 - i\xi_1 - j\xi_2 - k\xi_3} \tag{6b}$$

with magnitude and norm given by, respectively,

$$|q| = \sqrt{\varepsilon} = \sqrt{\xi_0^2 + \xi_1^2 + \xi_2^2 + \xi_3^2} = \sqrt{\varepsilon_0 + \varepsilon_1 + \varepsilon_2 + \varepsilon_3} \tag{7}$$

$$|q|^2 = q\bar{q} = \varepsilon = \varepsilon_0 + \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \tag{8}$$

It was shown in [12] that (4), (5) and (8) replicate the invariant mass associated with the Lorentz transformation of momenta in Euclidean spacetime. This observation suggests that the Lorentz group has a deep dynamical origin, tied to the onset of the MFM in proximity to the Fermi scale.

It has been long known that quaternions are able to reproduce the formal structure of Dirac's theory, of the electroweak sector as well as of GR [16 - 21]. Exploiting this fact and appealing to a) the representation of quaternions in terms of Pauli matrices along with b) the quaternion formulation of Einstein's field equations [19 - 20], leads to the following connection between the *metric tensor* $g_{\mu\nu}$ and *dimensional quaternions* (6)

$$\sigma_4 g_{\mu\nu} = -\frac{1}{2}(q_\mu \overline{q_\nu} + q_\nu \overline{q_\mu}) \tag{9}$$

in which,

$$q^\mu = \xi_\nu^\mu \sigma^\nu = \begin{pmatrix} \xi_0^\mu + \xi_3^\mu & \xi_1^\mu - i\xi_2^\mu \\ \xi_1^\mu + i\xi_2^\mu & \xi_0^\mu - \xi_3^\mu \end{pmatrix} \tag{10}$$

and where the set of four Pauli matrices are listed as

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\ \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\ \sigma_4 = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix} \tag{11}$$

The components of the metric tensor derived from (9) assume the form

$$\boxed{g_{\mu\nu} = -\frac{1}{4} Tr\,(q_\mu \overline{q_\nu} + q_\nu \overline{q_\mu})} \tag{12}$$

Interestingly enough, (12) echoes the duality between fractional dynamics in Minkowski spacetime and gravitational physics in curved spacetime, as detailed in [2].

3. DM and the duality of Cantor Dust and classical gravity

The key premise of [22] is that DM is not an analog of particle phenomena but rather an outcome of gravitational nonlinearities manifest in GR equations. Self-interaction is inherent in Einstein's equations, it fails to be suppressed at low velocities and it becomes significant in the presence of strong gravity or large galactic masses. The relationship between self-interaction and gravitational non-linearities becomes explicit upon expanding the Einstein-Hilbert Lagrangian into a polynomial form, i.e. [22]

$$L_G = \frac{\sqrt{\det(g_{\mu\nu})}}{16\pi G} g_{\mu\nu} R^{\mu\nu} = \sum_{n=0}^{\infty} (16\pi GM)^{n/2} \left[\varphi^n \partial\varphi\partial\varphi \right] \tag{13}$$

Using (12), it is a straightforward (albeit tedious) exercise to reformulate (13) in terms of dimensional quaternions (6). The result of this exercise is that gravitational nonlinearities appear as an emergent property of the MFM.

It is apparent from this discussion that accounting for the content of [13-15] along with (12-13) hints to the dual aspects of DM as Cantor Dust and gravitational physics.

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